**Exercice 1 :**

$$A = 1 - \left(\frac{10^3}{2}\right)^2 = 1 - \frac{10^6}{4} = \frac{4-10^6}{4} = \frac{25(4-10^6)}{10^2}) = -249999 \in \mathbb{Z}$$

$$B = \frac{3}{2} + \frac{5}{2} \times \frac{4}{5} - \frac{9}{2} = \frac{3}{2} + \frac{4}{2} - \frac{9}{2} = -\frac{2}{2} = -1 \in \mathbb{Z}$$

$$C = \frac{5\sqrt{3}+\sqrt{27}}{4\sqrt{3}} = \frac{5\sqrt{3}+3\sqrt{3}}{4\sqrt{3}} = \frac{8\sqrt{3}}{4\sqrt{3}} = 2 \in \mathbb{N}$$

$$D = \frac{4\times 10^{-4}\times 5\times 10^2}{3\times 10^{-1}\times 5\times 10^7} = \frac{4}{3} \times \frac{10^{-2}}{10^6} = \frac{4}{3}\times 10^{-8} = \frac{4}{3.10^8} \in \mathbb{Q}$$

**Exercice 2 :**
Montrer que les nombres ci-dessous sont des entiers naturels.

$$E = \frac{2}{\sqrt{5}-2} - 2\sqrt{5} = \frac{2(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)} - 2\sqrt{5} = \frac{2\sqrt{5}+4}{5-4} - 2\sqrt{5} = 2\sqrt{5}+4-2\sqrt{5} = 4 \in \mathbb{N}$$

$$F = \frac{(3+\sqrt{5})^2}{2} - 3\sqrt{5} = \frac{9+6\sqrt{5}+5}{2} - 3\sqrt{5} = \frac{14+6\sqrt{5}}{2} - 3\sqrt{5} = \frac{2(7+3\sqrt{5})}{2} - 3\sqrt{5} = 7+3\sqrt{5}-3\sqrt{5} = 7 \in \mathbb{N}$$

$$G = \frac{\frac{1}{2}+\frac{1}{4}}{\frac{1}{2}-\frac{1}{4}} = \frac{\frac{2}{4}+\frac{1}{4}}{\frac{2}{4}-\frac{1}{4}} = \frac{\frac{3}{4}}{\frac{1}{4}} = \frac{3}{4}\times\frac{4}{1} = 3 \in \mathbb{N}$$

$H = 55 \times 0,0545\underline{454}$
On pose $x = 0,05454\underline{54}$ donc $100x = 5,4545\underline{54}$
donc $100x - x = 5,454\underline{54} - 0,054\underline{54}$
donc $99x = 5,4$ donc $x = \dfrac{54}{990} = \dfrac{3}{55}$
Donc $H = 55 \times \dfrac{3}{55} = 3 \in \mathbb{N}$

**Exercice 3 :**
On pose $x = 140720$

$$I = 140720^2 - 140719 \times 140721 = x^2 - (x-1)(x+1) = x^2 - (x^2-1) = x^2 - x^2 + 1 = 1$$

$$J = 140721^2 - 140719^2 = (x+1)^2 - (x-1)^2 = x^2+2x+1-x^2+2x-1 = 4x = 562840$$

$$J = \left(\frac{140721}{2}\right)^2 - \left(\frac{140719}{2}\right)^2 = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x+1}{2}\right)^2 = \frac{x^2+2x+1}{4} - \frac{x^2-2x+1}{4} = \frac{4x}{4} = x = 140720$$