

Exercice :

Pour chacune des fonctions ci-dessous, trouver l'ensemble de définition, l'ensemble de dérivation et la fonction dérivée.

$$f(x) = -2x^2 + 5x^3 - \sqrt{2}x + \pi \quad g(x) = 7(-2x - 1) + 3x^2 + 1 \quad h(x) = (-2x - 1)(3x^2 + 1)$$

$$w(x) = \frac{1}{3x^2 + 1}$$

$$i(x) = \sqrt{-x + 1}$$

$$j(x) = 2x - 1 + \frac{1}{2x - 1}$$

$$k(x) = x^2\sqrt{x - 1}$$

$$l(x) = \frac{3 - 4x}{3x - 4}$$

$$m(x) = \frac{x^2 - 1}{x^2 + 1}$$

$$n(x) = \frac{x^2 - 11x + 30}{x^2 - x}$$

$$o(x) = \frac{2 \sin x}{\cos x}$$

$$p(x) = \frac{\cos x}{\sin x}$$

$$q(x) = \sin x + \cos x$$

$$r(x) = \frac{\sin x}{\cos x}$$

$$v(x) = \frac{1}{\sqrt{x}} - 7x^3$$

$$w(x) = 3\sqrt{x} + \frac{3}{x^2} - \frac{5}{x}$$

$$z(x) = 4x^{10} - 5x^6 + \frac{1}{x^7}$$

Correction :

1. $D_f = \mathbb{R}$ $D_{f'} = \mathbb{R}$ $f'(x) = 15x^2 - 4x - \sqrt{2}$
2. $D_g = \mathbb{R}$ $D_{g'} = \mathbb{R}$ $g'(x) = -14 + 6x$
3. $D_h = \mathbb{R}$ $D_{h'} = \mathbb{R}$ $h'(x) = -18x^2 - 6x - 2$
4. $D_w = \mathbb{R}$ $D_{w'} = \mathbb{R}$ $w'(x) = \frac{-6x}{(3x^2 + 1)^2}$
5. $D_v = \mathbb{R}$ $D_{v'} = \mathbb{R}$ $v'(x) = 20(5x + 1)^3$
6. $D_i =] - \infty; 1[$ $D_{i'} =] - \infty; 1[$ $i'(x) = -\frac{1}{2\sqrt{-x+1}}$
7. $D_j = \mathbb{R} \setminus \left\{ \frac{1}{2} \right\}$ $D_{j'} = \mathbb{R} \setminus \left\{ \frac{1}{2} \right\}$ $j'(x) = \frac{8x^2 - 8x}{(2x - 1)^2}$
8. $D_k = [1; +\infty[$ $D_{k'} = [1; +\infty[$ $k'(x) = \frac{x(5x - 4)}{2\sqrt{x - 1}}$
9. $D_l = \mathbb{R} \setminus \left\{ \frac{4}{3} \right\}$ $D_{l'} = \mathbb{R} \setminus \left\{ \frac{4}{3} \right\}$ $l'(x) = \frac{7}{(3x - 4)^2}$
10. $D_m = \mathbb{R}$ $D_{m'} = \mathbb{R}$ $m'(x) = \frac{4x}{(x^2 + 1)^2}$
11. $D_n = \mathbb{R} \setminus \{0; 1\}$ $D_{n'} = \mathbb{R} \setminus \{0; 1\}$ $n'(x) = \frac{10(x^2 - 6x + 3)}{(x^2 - x)^2}$
12. $D_o = D_{o'} = \mathbb{R} \setminus \left\{ -\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi \text{ avec } k \in \mathbb{Z} \right\}$ $o'(x) = 2(1 + \tan^2 x) = \frac{2}{\cos^2 x}$
13. $D_p = D_{p'} = \mathbb{R} \setminus \{2k\pi; -\pi + 2k\pi \text{ avec } k \in \mathbb{Z}\}$ $p'(x) = -\frac{1}{\sin^2 x}$
14. $D_q = D_{q'} = \mathbb{R}$ $q'(x) = 2(1 - 2\sin^2 x) = 2(2\cos^2 x - 1)$
15. $D_r = D_{r'} = \mathbb{R}$ $r'(x) = 3\sin(-3x + 5)$
16. $D_s = \left] -\infty; \frac{5}{3} \right[$ $D_{s'} = \left] -\infty; \frac{5}{3} \right[$ $s'(x) = -\frac{3}{2\sqrt{-3x+5}}$
17. $D_t = \left[\frac{3}{2}; +\infty \right[$ $D_{t'} = \left[\frac{3}{2}; +\infty \right[$ $t'(x) = \frac{3(x - 1)}{\sqrt{2x - 3}}$
18. $D_u = D_{u'} = \mathbb{R}$ $u'(x) = \frac{x^2 - 2}{x^2}$
19. $D_v = D_{v'} =]0; +\infty[$ $v'(x) = -\frac{1 + 42x^3\sqrt{x}}{2x\sqrt{x}}$
20. $D_w = D_{w'} =]0; +\infty[$ $w'(x) = \frac{3}{2\sqrt{x}} - \frac{6}{x^3} + \frac{5}{x^2}$
21. $D_z = D_{z'} = \mathbb{R} \setminus \{0\}$ $z(x) = 40x^9 - 30x^5 - \frac{7}{x^8}$