Exercice :

Pour chacune des fonctions ci-dessous, trouver l'ensemble de définition, l'ensemble de dérivation et la fonction dérivée.

$f(x) = -2x^2 + 5x^3 - \sqrt{2}x + \pi$ $g(x) = 7(-2x-1) + 3x^2 + 1$ $h(x) = (-2x-1)(3x^2+1)$

$w(x) = \dfrac{1}{3x^2+1}$ $v(x) = (5x+1)^4$ $i(x) = \sqrt{-x+1}$

$j(x) = 2x - 1 + \dfrac{1}{2x-1}$ $k(x) = x^2\sqrt{x-1}$ $l(x) = \dfrac{3-4x}{3x-4}$

$m(x) = \dfrac{x^2-1}{x^2+1}$ $n(x) = \dfrac{x^2-11x+30}{x^2-x}$ $o(x) = \dfrac{2\sin x}{\cos x}$

$p(x) = \dfrac{\cos x}{\sin x}$ $q(x) = (\sin x + \cos x)^2$ $r(x) = \cos(-3x+5)$

$s(x) = \sqrt{-3x+5}$ $t(x) = x\sqrt{2x-3}$ $u(x) = \dfrac{1+(1-x)^2}{x}$

$v(x) = \dfrac{1}{\sqrt{x}} - 7x^3$ $w(x) = 3\sqrt{x} + \dfrac{3}{x^2} - \dfrac{5}{x}$ $z(x) = 4x^{10} - 5x^6 + \dfrac{1}{x^7}$

Correction :

1. $D_f = \mathbb{R}$ $D_{f'} = \mathbb{R}$ $f'(x) = 15x^2 - 4x - \sqrt{2}$
2. $D_g = \mathbb{R}$ $D_{g'} = \mathbb{R}$ $g'(x) = -14 + 6x$
3. $D_h = \mathbb{R}$ $D_{h'} = \mathbb{R}$ $h'(x) = -18x^2 - 6x - 2$
4. $D_w = \mathbb{R}$ $D_{w'} = \mathbb{R}$ $w'(x) = \dfrac{-6x}{(3x^2+1)^2}$
5. $D_v = \mathbb{R}$ $D_{v'} = \mathbb{R}$ $v'(x) = 20(5x+1)^3$
6. $D_i =]-\infty;1]$ $D_{i'} =]-\infty;1[$ $i'(x) = -\dfrac{1}{2\sqrt{-x+1}}$
7. $D_j = \mathbb{R} \setminus \left\{\dfrac{1}{2}\right\}$ $D_{j'} = \mathbb{R} \setminus \left\{\dfrac{1}{2}\right\}$ $j'(x) = \dfrac{8x^2-8x}{(2x-1)^2}$
8. $D_k = [1;+\infty[$ $D_{k'} =]1;+\infty[$ $k'(x) = \dfrac{x(5x-4)}{2\sqrt{x-1}}$
9. $D_l = \mathbb{R} \setminus \left\{\dfrac{4}{3}\right\}$ $D_{l'} = \mathbb{R} \setminus \left\{\dfrac{4}{3}\right\}$ $l'(x) = \dfrac{7}{(3x-4)^2}$
10. $D_m = \mathbb{R}$ $D_{m'} = \mathbb{R}$ $m'(x) = \dfrac{4x}{(x^2+1)^2}$
11. $D_n = \mathbb{R} \setminus \{0;1\}$ $D_{n'} = \mathbb{R} \setminus \{0;1\}$ $n'(x) = \dfrac{10(x^2-6x+3)}{(x^2-x)^2}$

12. $D_o = D_{o'} = \mathbb{R} \setminus \left\{-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi \text{ avec } k \in \mathbb{Z}\right\}$ $\quad o'(x) = 2(1 + \tan^2 x) = \frac{2}{\cos^2 x}$

13. $D_p = D_{p'} = \mathbb{R} \setminus \{2k\pi; -\pi + 2k\pi \text{ avec } k \in \mathbb{Z}\}$ $\quad p'(x) = -\frac{1}{\sin^2 x}$

14. $D_q = D_{q'} = \mathbb{R}$ $\quad q'(x) = 2(1 - 2\sin^2 x) = 2(2\cos^2 x - 1)$

15. $D_r = D_{r'} = \mathbb{R}$ $\quad r'(x) = 3\sin(-3x + 5)$

16. $D_s = \left]-\infty; \frac{5}{3}\right]$ $\quad D_{s'} = \left]-\infty; \frac{5}{3}\right[$ $\quad s'(x) = -\frac{3}{2\sqrt{-3x+5}}$

17. $D_t = \left[\frac{3}{2}; +\infty\right[$ $\quad D_{t'} = \left]\frac{3}{2}; +\infty\right[$ $\quad t'(x) = \frac{3(x-1)}{\sqrt{2x-3}}$

18. $D_u = D_{u'} = \mathbb{R}$ $\quad u'(x) = \frac{x^2 - 2}{x^2}$

19. $D_v = D_{v'} =]0; +\infty[$ $\quad v'(x) = -\frac{1 + 42x^3\sqrt{x}}{2x\sqrt{x}}$

20. $D_w = D_{w'} =]0; +\infty[$ $\quad w'(x) = \frac{3}{2\sqrt{x}} - \frac{6}{x^3} + \frac{5}{x^2}$

21. $D_z = D_{z'} = \mathbb{R} \setminus \{0\}$ $\quad z(x) = 40x^9 - 30x^5 - \frac{7}{x^8}$